1. Let $V$ be a vector space with a spanning set $(\vec{x}, \vec{y}, \vec{z})$. If $\vec{u}, \vec{v}, \vec{w}, \vec{p} \in V$, prove $\vec{u}, \vec{v}, \vec{w}, \vec{p}$ are LD.

**Proof.** Assume $\vec{u}, \vec{v}, \vec{w}, \vec{p}$ are LI. Then, by Corollary 2.13, $(\vec{u}, \vec{v}, \vec{w}, \vec{p})$ can be expanded to a basis of $V$.

$\therefore \dim(V) \geq 4,$
$\therefore \{\vec{x}, \vec{y}, \vec{z}\}$ spans $V$, so it can be shrunk to a basis
$\therefore \dim(V) \leq 3,$ a contradiction to $\dim(V) \geq 4$
$\therefore \vec{u}, \vec{v}, \vec{w}, \vec{p}$ are LD. QED

2. A vector space $V$ is spaned by vectors $\vec{u}, \vec{v}, \vec{w}, \vec{p}, \vec{q}$. If $\dim(V) = 5$, prove $\vec{u}, \vec{v}, \vec{w}, \vec{p}, \vec{q}$ are LI.

**Proof.** Assume $\vec{u}, \vec{v}, \vec{w}, \vec{p}, \vec{q}$ are LD. Then they are not a basis.

$\therefore \{\vec{u}, \vec{v}, \vec{w}, \vec{x}, \vec{p}, \vec{q}\}$ is a spanning set of $V$,
$\therefore$ It can be shrunk to a basis of $V$.
$\therefore \dim(V) < 5,$ which contradict to $\dim(V) = 5$.
$\therefore \vec{u}, \vec{v}, \vec{w}, \vec{x}$ are LI. QED.

3. Extra credit: If $\vec{u}, \vec{v}, \vec{w}$ form a basis for a vector space, prove $a \cdot \vec{u} + \vec{w} \neq \vec{0}$ for any $a$.

**Proof.** Assume $a \cdot \vec{u} + \vec{w} = \vec{0}$.

Namely $a \cdot \vec{u} + 0 \cdot \vec{v} + 1 \cdot \vec{w} = a \cdot \vec{u} + \vec{w} = \vec{0}$ and $a, 0, 1$ are nontrivial.

$\therefore \vec{u}, \vec{v}, \vec{w}$ are LD, which is a contradiction to $\{\vec{u}, \vec{v}, \vec{w}\}$ being a basis (i.e. LI and spanning).

$\therefore a \cdot \vec{u} + \vec{v} \neq \vec{0}$. QED.