Homomorphism Exercises II

LI = Linear independent
LD = Linear dependent
LC = Linear combination
LT = Homomorphism
∃ = there is, there are

1. If $L$ is a LT and $L(3\bar{u}), L(-\bar{v}), L(5\bar{w})$ are LI, prove $\bar{u}, \bar{v}, \bar{w}$ are LI.

2. If $h$ is a LT and $\bar{u}, \bar{v}, \bar{w}$ are LD, prove $h(2\bar{u}), h(3\bar{v}), h(4\bar{w})$ are LD.

3. If $h: V \to W$ is a LT, $\dim(V) = 5$, $\dim(h^{-1}(\bar{0})) = 2$, $R(h)$ is spanned by $\{h(\bar{u}), h(\bar{v}), h(\bar{w})\}$. Prove $\{h(\bar{u}), h(\bar{v}), h(\bar{w})\}$ is a basis of $R(h)$.

4. Assume $h: V \to W$ is a LT and there is no $\bar{z} \neq \bar{0}$ such that $h(\bar{z}) = \bar{0}$. If $\{\bar{u}, \bar{v}, \bar{w}\}$ is a basis for $V$, prove $h(\bar{u}), h(\bar{v}), h(\bar{w})$ are LI.

5. Assume $h: V \to W$ is a LT and $\{\bar{u}, \bar{v}, \bar{w}\}$ is a basis for $V$. If $h(\bar{u}), h(\bar{v}), h(\bar{w})$ are LD, prove $\exists \bar{z} \neq \bar{0}$ such that $h(\bar{z}) = \bar{0}$.

6. Prove the following statement is false: Assume $h: V \to W$ is a LT. If $\{\bar{u}, \bar{v}\}$ is a basis for $V$, then $\{h(\bar{u}), h(\bar{v})\}$ is a basis for $R(h)$.

7. Prove the following statement is false: Assume $h: V \to W$ is a LT. If $\{h(\bar{u}), h(\bar{v})\}$ is a basis for $R(h)$, then $\{\bar{u}, \bar{v}\}$ is a basis for $V$.

8. Assume $h: V \to W$ is a LT and $\exists \bar{y} \neq \bar{z}$ such that $h(\bar{y}) = h(\bar{z})$. If $\{\bar{u}, \bar{v}, \bar{w}\}$ is a basis for $V$, prove $h(\bar{u}), h(\bar{v}), h(\bar{w})$ are LI.

9. Assume $h: V \to W$ is a LT and $\bar{y} \neq \bar{z}$. If $\{\bar{u}, \bar{v}, \bar{w}\}$ is a basis for $V$ and if $h(\bar{u}), h(\bar{v}), h(\bar{w})$ are LI, prove $h(\bar{y}) \neq h(\bar{z})$.

10. Assume $h: V \to W$ is a LT with $\dim(W) = 3$. If $\bar{u}, \bar{v}, \bar{w}, \bar{x} \in V$, prove $h(\bar{u}), h(\bar{v}), h(\bar{w}), h(\bar{x})$ are LD.