1. Prove the following set is a vector space

\[ V = \{ a + b \cdot x + c \cdot x^2 \mid c = a + b, \quad \text{and} \quad a, b, c \in \mathbb{R} \} \]

**Proof.** Let \( p = p_1 + p_2 \cdot x + p_3 \cdot x^2 \in V, \quad q = q_1 + q_2 \cdot x + q_3 \cdot x^2 \in V, \quad r \in \mathbb{R} \).

Then \( p + q = (p_1 + p_2 \cdot x + p_3 \cdot x^2) + (q_1 + q_2 \cdot x + q_3 \cdot x^2) = (p_1 + q_1) + (p_2 + q_2) \cdot x + (p_3 + q_3) \cdot x^2 \).

\[ r \cdot p = r \cdot (p_1 + p_2 \cdot x + p_3 \cdot x^2) = (r \cdot p_1) + (r \cdot p_2) \cdot x + (r \cdot p_3) \cdot x^2 \]

\[ \therefore p, q \in V \]

\[ \therefore p_3 + q_3 = (p_1 + p_2) + (q_1 + q_2) = (p_1 + q_1) + (p_2 + q_2) \]

\[ r \cdot p_3 = r \cdot (p_1 + p_2) = (r \cdot p_1) + (r \cdot p_2) \]

\[ \therefore V \text{ is a vector space. QED} \]

2. Prove the following set is a vector space

\[ W = \left\{ \begin{bmatrix} x & 0 \\ y & x + y \end{bmatrix} \mid x, y \in \mathbb{R} \right\} \]

**Proof.** Let \( A \in W, \quad B \in W, \quad r \in \mathbb{R} \)

\[ \therefore A, B \in W \]

\[ \therefore \text{there are } a, b, u, v \in \mathbb{R} \text{ such that } A = \begin{bmatrix} a & 0 \\ b & a + b \end{bmatrix}, \quad B = \begin{bmatrix} u & 0 \\ v & u + v \end{bmatrix} \]

\[ \therefore A + B = \begin{bmatrix} a + u & 0 + 0 \\ b + v & (a + b) + (u + v) \end{bmatrix} = \begin{bmatrix} a + u & 0 \\ b + v & (a + u) + (b + v) \end{bmatrix} = \begin{bmatrix} x & 0 \\ y & x + y \end{bmatrix} \]

where \( x = a + u, \quad y = b + v \)

\[ \therefore A + B \in W \]

Also \( r \cdot A = r \cdot \begin{bmatrix} a & 0 \\ b & a + b \end{bmatrix} = \begin{bmatrix} r \cdot a & 0 \\ r \cdot b & r \cdot (a + b) \end{bmatrix} = \begin{bmatrix} r \cdot a & 0 \\ r \cdot b + (r \cdot a) + (r \cdot b) \end{bmatrix} = \begin{bmatrix} x & 0 \\ y & x + y \end{bmatrix} \)

for \( x = r \cdot a, \quad y = r \cdot b \)

\[ \therefore W \text{ is a vector space.} \]