Problem 1 (20 points)

Write a program that, for input \(x_0, y_0, n\), generates sequences \(\{x_0, x_1, \ldots, x_n\}\) and \(\{y_0, y_1, \ldots, y_n\}\) by the iteration

\[
\begin{align*}
x_{k+1} &= 1 - y_k + \lfloor x_k \rfloor \\
y_{k+1} &= x_k
\end{align*}
\]

Use input \(x_0 = 1.5, y_0 = 2.13\) to generate and plot the set of points \(\{(x_j, y_j) \mid j = 0, 1, \ldots, n\}\). The graph is called the Gingerbreadman map.

Solution:

```plaintext
> read("c:/temp/ginger.mpl");
Ginger := proc(x0, y0, n) 
    local x, y, k;
    x[0], y[0] := x0, y0;
    for k from 0 to n do
        x[k+1] := 1 - y[k] + abs(x[k]);
        y[k+1] := x[k];
    end do;
    return x, y;
end proc;
~
~
-- INSERT --  3,16  Bot
```

> (1.1)
return x, y
end proc

> n := 2000:
> x, y := Ginger(1.5, 2.13, n):
> p := [seq([x[k], y[k]], k = 0..n)]:
> plot(p, style=point);

\textbf{Problem 2 (20 points)}

Write a program to calculate the following sum and use WYSIWYG method to verify your program for $n = 5$

$$\sum_{k=1}^{n} \frac{(-1)^{k+1} \cdot \sum_{i=1}^{k} (2 \cdot i - 1)^2}{\prod_{j=1}^{k} (2 \cdot j)}$$
\[
\sum_{k=1}^{5} \frac{(-1)^k + 1 \cdot \sum_{i=1}^{k} (2i - 1)^2}{9 + \prod_{j=1}^{k}(2j)}
\]

\[
\frac{97033009}{1791490107}
\]
Problem 3 (20 points)

For any input positive number $M$, the sum

$$\sum_{k=2}^{n} \frac{1}{\sqrt{k} \cdot \ln(k)}$$

will be larger than $M$ when $n$ is large enough. Use while-do to calculate the sum and stop when the sum is larger than $M$. Output $n$.

Solution:

```plaintext
read("j:/340/sum4.mpl"): sum4(3);
```

```
10
```

```
sum4(5.5);
```

```
50
```

Problem 4 (20 points)

Write a program that, for input numbers $a$ and $b$ output the number $s$ according to the following formula/diagram:
\[ s = \begin{cases} a + 2b & \text{if } a < 5 \text{ and } b > 0 \\ 2a + b & \text{if } a < 5 \text{ and } b < 2 \\ 3 + ab & \text{if } a < 5 \text{ and } b = 2 \\ 2 - a + b & \text{if } a < 5 \text{ and } b > 2 \\ 3a - 5b & \text{if } a \geq 5 \text{ or } b \leq 0 \end{cases} \]
Problem 5. (20 points)
For input a vector \( x = [x_1, \ldots, x_n] \) and \( y = [y_1, \ldots, y_n] \), write a program to carry out the following computation and output the result:

\[
\sqrt{\frac{1}{n} \sum_{k=1}^{n} (x_k^2 - y_k^2)}
\]

Use random vectors to test your program.

Solution:

\[
\text{read("j:/340/sum5.mpl");}
\]

\[
\text{x := LinearAlgebra:-RandomVector[row](9);}
\]

\[
\text{y := LinearAlgebra:-RandomVector[row](9);}
\]

\[
\text{sum5(x,y);}
\]

\[
> \text{read("j:/340/sum5.mpl");}
\]

\[
\text{sum5} := \text{proc} (x::\text{Vector}, y::\text{Vector})
\]

\[
\text{local n, k, s;}
\]

\[
\text{n := LinearAlgebra:-Dimension(x);}
\]

\[
\text{s := 0;}
\]

\[
\text{for k from 1 to n do}
\]

\[
\text{s := s + abs(x[k]^2 - y[k]^2);}
\]

\[
\text{end do;}
\]

\[
\text{return sqrt(s/n);}
\]

\[
\text{end proc;}
\]

\[
> \text{x := LinearAlgebra:-RandomVector[row](9);}
\]

\[
\text{x := [25 94 12 -2 50 10 -16 -9 -50]}
\]

\[
> \text{y := LinearAlgebra:-RandomVector[row](9);}
\]

\[
\text{y := [20 -61 -48 77 31 -50 -80 43]}
\]

\[
> \text{sum5(x,y);}
\]

\[
\frac{1}{3} \sqrt{25919}
\]
Problem 6. (Extra 15 points)

For an input vector \( x = [x_1, \ldots, x_n] \) and a real number \( t \), we want to know the index \( k \) at which \( s = |x_k - t| \) is smallest among \( |x_1 - t|, |x_2 - t|, \ldots, |x_n - t| \). Write a program to carry out the search and output \( k \) and \( s \).

```
> read("j:/340/nearest.mpl");
> x := LinearAlgebra:-RandomVector[row](9);
    x := Matrix(9, 1, [ -33, -68, -67, 22, 14, 16, 99, 99, 60 ])
> Nearest(x,16.9);
6, 0.9
```